# NAG Toolbox for MATLAB

# g01fb

## 1 Purpose

g01fb returns the deviate associated with the given tail probability of Student's $t$-distribution with real degrees of freedom, via the function name.

## 2 Syntax

```
[result, ifail] = g01fb(tail, p, df)
```

## 3 Description

The deviate, $t_p$ associated with the lower tail probability, $p$, of the Student's $t$-distribution with $\nu$ degrees of freedom is defined as the solution to

$$P\left(T < t_p : \nu\right) = p = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \int_{-\infty}^{t_p} \left(1 + \frac{T^2}{\nu}\right)^{-(\nu+1)/2} dT, \qquad \nu \geq 1;\ -\infty < t_p < \infty.$$

For $\nu = 1$ or 2 the integral equation is easily solved for $t_p$.

For other values of $\nu < 3$ a transformation to the beta distribution is used and the result obtained from g01fe.

For $\nu \geq 3$ an inverse asymptotic expansion of Cornish–Fisher type is used. The algorithm is described by Hill 1970.

## 4 References

Hastings N A J and Peacock J B 1975 *Statistical Distributions* Butterworths

Hill G W 1970 Student's $t$-distribution *Comm. ACM* **13** 617–619

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **tail – string**

Indicates which tail the supplied probability represents.

**tail** = 'U'

The upper tail probability, i.e., $P\left(T \geq t_p : \nu\right)$.

**tail** = 'S'

The two tail (significance level) probability, i.e., $P\left(T \geq \left|t_p\right| : \nu\right) + P\left(T \leq -\left|t_p\right| : \nu\right)$.

**tail** = 'C'

The two tail (confidence interval) probability, i.e., $P\left(T \leq \left|t_p\right| : \nu\right) - P\left(T \leq -\left|t_p\right| : \nu\right)$.

**tail** = 'L'

The lower tail probability, i.e., $P\left(T \leq t_p : \nu\right)$.

*Constraint*: **tail** = 'U', 'S', 'C' or 'L'.

2: **p – double scalar**

$p$, the probability from the required Student's $t$-distribution as defined by **tail**.

*Constraint*: $0.0 < \mathbf{p} < 1.0$.

3: **df – double scalar**

$\nu$, the degrees of freedom of the Student's $t$-distribution.

*Constraint*: $\mathbf{df} \geq 1.0$.

### 5.2 Optional Input Parameters

None.

### 5.3 Input Parameters Omitted from the MATLAB Interface

None.

### 5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

**Note**: g01fb may return useful information for one or more of the following detected errors or warnings.

If **ifail** = 1, 2, 3 or 4 on exit, then g01fb returns zero.

**ifail** = 1

On entry, **tail** $\neq$ 'U', 'S', 'C' or 'L'.

**ifail** = 2

On entry, $\mathbf{p} \leq 0.0$,
or $\mathbf{p} \geq 1.0$.

**ifail** = 3

On entry, $\mathbf{df} < 1.0$.

**ifail** = 4

The solution is too close to zero to be determined accurately. This error will only occur when $\mathbf{df} = 1.0$. The returned value of zero will be a good approximation in terms of absolute value but will have a poor relative precision.

**ifail** = 5

Convergence in the calculation of the inverse beta value was not achieved. However, the result should be a reasonable approximation to the correct value.

## 7 Accuracy

The results should be accurate to five significant digits, for most parameter values. The error behaviour for various parameter values is discussed in Hill 1970.

## 8 Further Comments

The value $t_p$ may be calculated by using the transformation described in Section 3 and using g01fe. g01fb allows you to set the required accuracy.

## 9 Example

```
tail = 'S';
p = 0.01;
df = 20;
[result, ifail] = g01fb(tail, p, df)
```

```
result =
    2.8453
ifail =
           0
```